The contribution of spin torque to spin Hall coefficient and spin motive force in spin-orbit coupling system

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Abstract. We derive rigorously the relativistic angular momentum conservation equation by means of quantum electrodynamics. The novel nonrelativistic spin current and torque in the spin-orbit coupling system, up to the order of $1/c^4$, are exactly investigated by using Foldy-Wouthuysen transformation. We find a perfect spin Hall coefficient including the contribution of spin torque dipole. A novel spin motive force, analogue to the Lorentz force, is also obtained for understanding of the spin Hall effect.

1. Introduction

Spintronics has become a fast developing field since it developed. The transport concerned aspect of the carriers' spin degree and the spin Hall effect [1-3] were paid a lot of attention recently. In order to describe the spin transport properly, the definition of spin current was discussed and various theories of spin current have been established [4, 5]. In a traditional review, the spin current was presented in terms of an anticommutator of the velocity and the spin, $(1/2)\varphi^{+}\{\mathbf{v},\mathbf{s}\}\varphi$. However, under such a definition one of problem is that there is not a conjugate spin force to link the spin current. Therefore, the Onsager relation can not be established [6]. Furthermore, because the spin has its own dynamics in its Hilbert space, the current with both spin and spatial degree is not conserved due to the spin-orbit coupling. With the consideration of a spin torque, a source in the spin continuity equation can be achieved. Previous investigations in the spin torque depend on the spin relaxation time [7-12]. To our knowledge, an explicit torque beyond of approximation of spin relaxation time has not been established yet.

In the studies of spin Hall effect, the experiments and theories focus on the spin Hall coefficient σ_{SH} [4, 5, 13-31]. In comparison of Ohm's law in electronics responded to the applied electric field a spin current j_s^{kl} is generated, $j_s^{kl} = \sigma_{SH}\varepsilon^{lkm}E^m$ [4]. Recent studies shew that the spin Hall coefficient σ_{SH} not only includes the contribution of the conventional spin current, but also the torque dipoles which are contained in semiconductor models with the effect of disorder [6, 32]. However, those contributions from the torque dipoles have not been clearly found yet.

Based on the above considerations, the consistency of quantum electrodynamics and Noether's theorem in the derivation of the exact conservation equation for the relativistic angular momentum was suggested [33, 34]. It is found that the spin current including a correction is different from the traditional definition. In the application the spin Hall conductivity σ_{SH} involved the correction can be obtained. Under the requirement of the Onsager relation the spin force is found to relate to the spin Hall coefficient, therefore, relate the topological aspect of systems with the spin-orbit coupling.

2. Spin continuity equation

Let us firstly consider the relativistic Lagrangian with Dirac fields Ψ and $\bar{\Psi}$ coupled to an electromagnetic field A^{μ}, $\mathcal{L} = \mathcal{L}_{\rm D} + \mathcal{L}_{\rm em} + \mathcal{L}_{\rm int}$, where $\mathcal{L}_{\rm D} = \bar{\Psi}(i\hbar c\gamma^{\mu}\partial_{\mu} - mc^2)\Psi$ describes the free Dirac fields of spin 1/2, $\mathcal{L}_{\rm em} = -(1/4)F^{\mu\nu}F_{\mu\nu}$ is the Lagrangian of electromagnetic field, where $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$, the interaction between Dirac fields and electromagnetic field is given by $\mathcal{L}_{\rm int} = -e\bar{\Psi}\gamma^{\mu}A_{\mu}\Psi$, and the four-vector γ^{μ} is represented as $\gamma^{\mu} = (\gamma^0, \gamma)$ in terms of Pauli matrices σ.

The energy-momentum tensor of a gauge invariant form is found to be $\theta^{\mu\nu} = \theta^{\mu\nu}_{\rm D} + \theta^{\mu\nu}_{\rm em} + \theta^{\mu\nu}_{\rm int}$, where $\theta^{\mu\nu}_{\rm D} = \bar{\Psi}i\hbar c\gamma^{\mu}\partial^{\nu}\Psi - g^{\mu\nu}\mathcal{L}_{\rm D}$, $\theta^{\mu\nu}_{\rm em} = -F^{\mu\sigma}\partial^{\nu}A_{\sigma} - g^{\mu\nu}\mathcal{L}_{\rm em}$, and $\theta^{\mu\nu}_{\rm int} = -g^{\mu\nu}\mathcal{L}_{\rm int}$. Here $g^{\mu\nu} = g_{\mu\nu}$ is the metric tensor with $g^{00} = 1$, $g^{ii} = -1$ $(i = 1, 2, 3)$

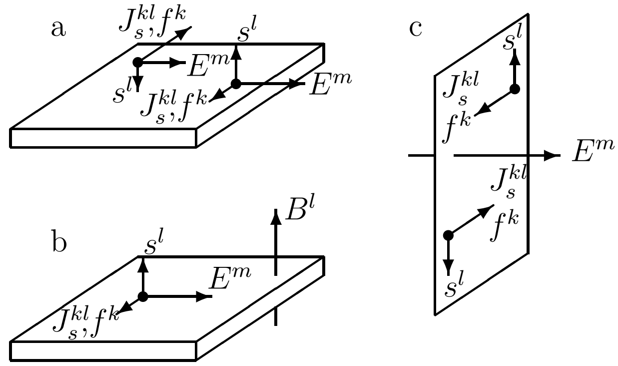

Figure 1. The spin current J_s^{kl} and the spin motive force f^k via the spin $\mathbf{s}$ and the electric field $\mathbf{E}$, where J_s^{kl} represents the current of the l component s^l of the spin along the direction k. (a) the spin current J_s^{kl} and the spin motive force f^k in the spin-orbit coupling system without an external magnetic field, where E^m, s^l and J_s^{kl} (or f^k) satisfy the right-hand rule; (b) the spin current and the spin motive force in the spin-orbit coupling system under an external magnetic field $\mathbf{B}$ along the l direction; (c) the spin current and the spin motive force in the two-dimensional Rashba spin-orbit coupling system.

and $g^{\mu\nu} = 0$ $(\mu, \nu = 0, 1, 2, 3, \mu \neq \nu)$. This energy-momentum tensor satisfies the conservation law, i.e., $\partial_\mu \theta^{\mu\nu} = 0$. With the tensor the angular momentum tensor can be written in the form of $M^{\alpha\mu\nu} = s^{\alpha\mu\nu} + l^{\alpha\mu\nu}$. Here $l^{\alpha\mu\nu} = x^\mu \theta^{\alpha\nu} - x^\nu \theta^{\alpha\mu}$ is the orbital angular momentum tensor and $s^{\alpha\mu\nu} = s_{\mathrm{D}}^{\alpha\mu\nu} + s_{\mathrm{em}}^{\alpha\mu\nu}$ is spin angular momentum tensor, where $s_{\mathrm{D}}^{\alpha\mu\nu} = (\partial\mathcal{L}/\partial\partial_\alpha\Psi)\, I_{\mathrm{D}}^{\mu\nu}\Psi$ and $s_{\mathrm{em}}^{\alpha\mu\nu} = (\partial\mathcal{L}/\partial\partial_\alpha A_\sigma)\,(I_{\mathrm{em}}^{\mu\nu})_{\sigma\rho}A^\rho$. Considering the notations $I_{\mathrm{D}}^{\mu\nu} = -i\sigma^{\mu\nu}/2$ and $(I_{\mathrm{em}}^{\mu\nu})_{\sigma\rho} = g_\sigma^\mu g_\rho^\nu - g_\rho^\mu g_\sigma^\nu$, it is found $s_{\mathrm{D}}^{\alpha\mu\nu} = i\,(\hbar c/4)\,\bar{\Psi}\gamma^\alpha[\gamma^\mu, \gamma^\nu]\Psi$ and $s_{\mathrm{em}}^{\alpha\mu\nu} = A^\mu F^{\alpha\nu} - A^\nu F^{\alpha\mu}$. The corresponding conservation law for the total angular momentum is $\partial_\alpha M^{\alpha\mu\nu} = 0$.

In order to obtain the nonrelativistic form of the conservation law, the Foldy-Wouthuysen transformation is used in the following calculations up to $1/c^4$. The nonrelativistic wave function is written in terms of a transformation on the relativistic wave function Ψ, $\Psi'' = \exp[is'(\alpha)]\exp[is(\alpha)]\,\Psi$, where the operators in the exponential are $is(\alpha) \equiv (\beta/2mc)\,\alpha \cdot \pi$ and $is'(\alpha) \equiv (i\hbar e/4m^2c^3)\,\alpha \cdot \mathbf{E}$. Here $\mathbf{E}$ is the electric field intensity. Correspondingly the wave function is written in the form as $\Psi'' = (\varphi'', \chi'')^T$, where $\varphi'' = \left[1 - s(\sigma)^2/2\beta^2\right]\varphi$ and $\chi'' = \left[is'(\sigma) - i\,(E - e\phi)\,s(\sigma)/2mc^2\beta - is(\sigma)^3/3\beta^3\right]\varphi$. Introducing a notion $\eta = i\hbar e\sigma \cdot \mathbf{E} - (E - e\phi)\sigma \cdot \pi - (\sigma \cdot \pi)^3/6m$, χ'' is presented as $\chi'' = (\eta/4m^2c^3)\,\varphi$. With the help of formula $e^{is(\alpha)}\hat{O}e^{-is(\alpha)} = \hat{O} + [is, \hat{O}] + [is, [is, \hat{O}]]/2 + \cdots + [is, [is, \cdots, [is, \hat{O}]\cdots]]/n! + \cdots$ and let $\hat{O}$ be M^{0ij} and M^{kij}, the continuity equation for the nonrelativistic electronic spin can be obtained. The nonrelatistivic form of angular momentum conservation law reads

$$\frac{\partial}{\partial t}\rho_s^l + \nabla^k j_s^{kl} = T^l, \tag{1}$$

where $j_s^{kl} = (\hbar/4m)\,\varphi^\dagger\left\{\pi^k, \sigma^l\right\}\varphi$ is the traditional spin current, which represents the

current of the l component of the spin along the direction k. Here we have written the wave function φ'' as φ for the convenient. The spin density ρ_s^l is obtained as

$$\rho_s^l = \frac{\hbar}{2}\varphi^+\sigma^l\varphi + \frac{\hbar}{4m^2c^2}\varphi^+(\pi^l\sigma\cdot\pi - \pi^2\sigma^l)\varphi + \frac{\hbar^2 e}{8m^2c^3}\varphi^+(3B^l - \sigma^l\sigma\cdot\mathbf{B})\varphi + \frac{i\hbar}{8m^3c^4}\varphi^+[(\sigma\times\pi)^l\eta - \eta^+(\sigma\times\pi)^l]\varphi, \quad (2)$$

where magnetic field $\mathbf{B}$ is written out evidently. The first term in Eq. (2) is nothing but a traditional spin density. The second term can be written as $(\hbar/4m^2c^2)\,\varphi^+\pi\times(\pi\times\sigma)\varphi$, which indicates its generation from the spin-orbit coupling. The interaction between the intrinsic magnetic moment and the external magnetic field is given by the third term. The last term gives a small correction in the order of $1/c^4$.

Now let us analysis the right hand of Eq. (1), named the spin torque density T^l. Up to the same order of the nonrelativistic approximation, it is found

$$\begin{aligned} T^l = {} & \nabla^k\{\frac{i\hbar}{2m}\varphi^+\sigma^k(\sigma\times\pi)^l\varphi\} + \frac{i\hbar e}{2mc}\varphi^+[\sigma^l\sigma\cdot\mathbf{B} - B^l]\varphi \\ & - \frac{\hbar e}{4m^2c^2}\varphi^+\{\hbar[\nabla(\mathbf{E}\cdot\sigma)\times\sigma]^l + 2\sigma^l\pi\cdot\mathbf{E} - 2\sigma\cdot\pi\ E^l\}\varphi \\ & + \frac{\hbar^2 e}{4m^2c^2}\nabla^k\{\varphi^+[\sigma^k(\sigma\times\mathbf{E})^l + (\sigma\times\mathbf{E})^k\sigma^l]\varphi\} \\ & - \frac{1}{32m^4c^4}\varphi^+(\eta^+\{\sigma\cdot\pi,\{\sigma\cdot\pi,(\pi\times\sigma)^l\}\} + \{\sigma\cdot\pi,\{\sigma\cdot\pi,(\pi\times\sigma)^l\}\}\eta)\varphi \\ & + \frac{\hbar}{64m^4c^4}\nabla^k[\varphi^+(\eta^+\{\sigma\cdot\pi,\{\sigma\cdot\pi,\sigma^k\sigma^l\}\} + \{\sigma\cdot\pi,\{\sigma\cdot\pi,\sigma^k\sigma^l\}\}\eta)\varphi]. \end{aligned} \quad (3)$$

Besides of the relativistic correction up to the order of $1/c^4$, the contributions from the spin-orbit coupling and its nonrelativistic correction are presented by the first and the forth terms. The second term corresponds to the interaction of intrinsic magnetic moment and external magnetic field. The effect from the couplings among the orbit and the spin to the electric field is given in the third term.

Previous discussion of spin Hall effect was given in the case of absence of the magnetic field $\mathbf{B}$. In general, to extend the cases for the ferromagnet or the system under the external magnetic field, the magnetic field is remained in the follows and demonstrated the effect of magnetic field on the spin Hall effect. Considering an external magnetic field along the direction of the spin, one state of the spin polarization is left and all spin transport processes in the presence of both the electric field E^m and the magnetic field B^l are shown in Fig. 1(b). The corresponding the spin motive force and the spin Hall coefficient can be obtained. It is worth to point out that the previous spin current does not contain the contribution of spin torque dipole [6]. When the torque density is written in the form of a divergence of a torque dipole $T^l = -\nabla^k P_T^{kl}$, where $P_T^{kl} = \int_v T^l dx^k$ is integrable, the spin current is found

$$J_s^{kl} = j_s^{kl} + P_T^{kl}, \quad (4)$$

which includes the traditional current and a correction of the spin torque dipole. Eq. (4) can be written as a response equation $J_s^{kl} = \sigma_{sc}\varepsilon^{lkm}E^m$ in which σ_{sc} is the spin Hall

coefficient. Obviously, the spin current J_s^{kl} is vertical to the direction of the spin s^l and the electric field E^m. E^m, s^l, and J_s^{kl} satisfy the right-hand rule, as shown in Fig. 1(a).

Now the spin continuity equation (1) can be written as

$$\frac{\partial}{\partial t}\rho_s^l + \nabla^k J_s^{kl} = 0. \tag{5}$$

It implies that the spin current has a natural conjugate spin force. Therefore, the Onsager relation $\sigma_{sc}^{mk} = -\sigma_{cs}^{km}$ can be established under the time reversal symmetry to link the spin transport with other transport phenomena, such as the charge transport, where σ_{sc}^{mk} and σ_{cs}^{km} are the spin-charge and charge-spin conductivity tensors.

3. Spin Hall coefficient and spin motive force

We consider the divergence of the spin torque dipole as a product of a electric field and a coefficient $\chi^{lm}(\mathbf{q})$, $-iq^k P_T^{kl}(\mathbf{q}) = \chi^{lm}(\mathbf{q})E^m(\mathbf{q})$, with $\mathbf{q}$ being a finite wave vector. The more explicit form of the coefficient can be represented as follow

$$\begin{aligned}\chi^{lm} = &-\frac{\hbar}{2}\varepsilon^{lm'm}q^{m'}\frac{\sigma_e}{e} + \frac{i\hbar e}{2mc}\varphi^+(\mathbf{q})[(\sigma^l\sigma\cdot\mathbf{B} - B^l)/E^m]\varphi(\mathbf{q}) \\ &-\frac{\hbar e}{4m^2c^2}\varphi^+(\mathbf{q})(i\hbar q^{m'}\sigma^m\sigma^{n'}\varepsilon^{lm'n'} + 2\sigma^l\pi^m)\varphi(\mathbf{q}),\end{aligned} \tag{6}$$

where σ_e is the electric conductivity. The spin Hall coefficient σ_{sc} corresponding to our new spin current J_s^{kl} can be written as

$$\sigma_{sc} = \sigma_{SH}^0 + \sigma_{SH}^T, \tag{7}$$

where σ_{SH}^0 is the conventional spin Hall conductivity [4, 12], corresponding to the traditional spin current, σ_{SH}^T is the contribution of the spin torque dipole P_T^{kl}, and $\sigma_{SH}^T = Re\{i\partial\chi^{lm}(\mathbf{q})/\partial q^k\}_{\mathbf{q}=0}$. In some semiconductors with disorder the spin Hall coefficient is extremely different from the conventional one. We can evaluate the spin Hall coefficient σ_{SH}^T in the GaAs sample as follows: at room temperature, the carrier density of GaAs is $n \sim 10^{17}\mathrm{cm}^{-3}$, the mobility of carriers is $\mu \sim 350\mathrm{cm}^2/\mathrm{Vs}$, the conventional spin Hall coefficient is $\sigma_{SH}^0 \sim 16\Omega^{-1}\mathrm{cm}^{-1}$, $\sigma_{SH}^T \sim 5.6\Omega^{-1}\mathrm{cm}^{-1}$. For lower carrier density case, $n \sim 10^{16}\mathrm{cm}^{-3}$, $\mu \sim 400\mathrm{cm}^2/\mathrm{Vs}$, $\sigma_{SH}^0 \sim 7.3\Omega^{-1}\mathrm{cm}^{-1}$, σ_{SH}^T is estimated as $\sigma_{SH}^T \sim 0.64\Omega^{-1}\mathrm{cm}^{-1}$. As a kind of correction, σ_{SH}^T is one order smaller than the conventional spin Hall coefficient σ_{SH}^0. The general spin Hall coefficient σ_{sc} should include the conventional one σ_{SH}^0 and the correction σ_{SH}^T.

Now the Onsager relation and spin Hall coefficient have been found. The so-called spin force $\mathbf{F}_s$ can be calculated as $\mathbf{F}_s = (\mathbf{J}_c - \sigma_{cc}\mathbf{E})/\sigma_{cs}$, where σ_{cc} is the charge-charge conductivity tensor, and $\mathbf{J}_c$ is charge current [6]. Particularly, in Ref. [12], the spin force has a simple form as $F_s^m = J_c^k/\sigma_{cs}^{km}$ in the two-dimensional electron gas. From Onsager relation, $\sigma_{cs}^{km} = -\sigma_{sc}^{mk}$, the charge-spin tensor σ_{cs}^{km} can be obtained, and the intrinsic Hall current J_c^k in the k direction can be detected by experiments. However, the spin force can not be interpreted as a motive force of electron like the Lorentz force in Hall effect, and it has the same direction with the electric field E^m.

To interpret the spin Hall effect, we try to find a spin motive force f^k which has an analogy to the Lorentz force in the Hall effect. Here the spin motive force is vertical to the direction of the electric field and the spin, i.e., E^m, s^l and f^k satisfy the right-hand rule, as shown in Fig. 1(a). The discussion is based on the spin torque. The torque density T^l can be written as the form $T^l = \varepsilon^{lmk} r^m f^k = \chi^{lm} E^m$. After calculation, we obtain f^k as

$$f^k = \sigma_f^1 E^m + \sigma_f^2 \chi^{lm}, \tag{8}$$

where the spin motive force coefficients σ_f^1 and σ_f^2 are expressed as

$$\sigma_f^1 = \frac{1}{2} Re\{\varepsilon^{lmk} \nabla^m \chi^{lm}(\mathbf{r})\} \tag{9}$$

and

$$\sigma_f^2 = \frac{1}{2} Re\{\varepsilon^{lmk} \nabla^m E^m(\mathbf{r})\}. \tag{10}$$

In the case of the electric field being constant, σ_f^2 is zero. Here we have obtained the evident formula χ^{lm}, and the electric field E^m can be detected in experiments. Thus the spin motive force f^k is found. Assuming the mobility of the carriers in the GaAs sample with disorder is $\mu \sim 10^3 \mathrm{cm}^2/\mathrm{Vs}$ and the electric field is $\mathbf{E} \sim 10\mathrm{mV}/\mu\mathrm{m}$, we find the order-of-magnitude of the spin motive force $f^k \sim 10^{-20}\mathrm{eV}/\mu\mathrm{m}$. Obviously, this is an extremely weak quantity.

4. Application in the two-dimensional electron gas

We will discuss the properties of the spin motive force in the two-dimensional electron gas. The Dirac Hamiltonian of relativistic electron is $H = c\alpha \cdot \mathbf{P} + \beta mc^2$. Using the F-W transformation, the nonrelativistic limit of the Dirac Hamiltonian is $H = \beta(mc^2 + \pi^2/2m - \pi^4/8m^3c^2) + e\phi - (\hbar e/2mc)\, \beta\sigma \cdot \mathbf{B} - \left(\hbar^2 e/8m^2c^2\right) \nabla \cdot \mathbf{E} - i \left(\hbar^2 e/8m^2c^2\right) \sigma \cdot (\nabla \times \mathbf{E}) - (\hbar e/4m^2c^2)\, \sigma \cdot (\mathbf{E} \times \mathbf{P})$, where ϕ is the electric potential [34]. In the two-dimensional electron gas, $\mathbf{E} = (0, 0, E^m)$, $\sigma = (\sigma^l, \sigma^k, \sigma^m)$, $\mathbf{P} = (P^l, P^k, 0)$, and $\mathbf{B} = 0$, the nonrelativistic Hamiltonian can be written as $H = \mathbf{P}^2/2m - \lambda(P^k\sigma^l - P^l\sigma^k)$, this is the Rashba Hamiltonian, where the coupling parameter $\lambda = (\hbar e/4m^2c^2)\, E^m$ [35].

In the two-dimensional electron gas, the formula χ^{lm} has a simple form, $\chi^{lm} = i\,(\hbar/2e)\, \varepsilon^{lm'm} \nabla^{m'} \sigma_e - (\hbar e/4m^2c^2)\, \varphi^+(\hbar \nabla^{m'} \sigma^m \sigma^{n'} \varepsilon^{lm'n'} + 2\sigma^l \pi^m)\varphi$. Thus the spin motive force can be represented as $f^k = \left(\varepsilon^{lkm} \hbar e/8m^2c^2\right) \nabla^m [\varphi^+(\hbar \nabla^{m'} \sigma^m \sigma^{n'} \varepsilon^{lm'n'} + 2\sigma^l \pi^m)\varphi] E^m$. The spin motive force f^k is nonzero, and it induces the spin current, so the spin Hall effect can be observed in experiments in the two-dimensional electron gas. In this case, f^k should be vertical to the spin s^l and electric field E^m, as shown in Fig. 1(c). In Ref. [36], the author introduced a spin transverse force which is perpendicular to the spin current. On the contrary, our spin motive force is parallel to the spin current. So it can be used to better understand the mechanism of the spin Hall effect.

In conclusion, we induce the spin continuity equation from the angular momentum conservation law with spin-orbit coupling. Our results naturally include a correction to

the traditional spin current. The correction could be considered as a spin torque dipole, so there is a conjugate force linking the spin current, and the Onsager relation can be established. A perfect spin Hall coefficient corresponding to the new spin current is conformed. Furthermore, the magnitude of the spin Hall coefficient is evaluated. From the explicit spin torque, we introduce a spin motive force having the same direction with the spin current to better understand the spin Hall effect. We find a novel right-hand rule among the electric field, the spin and spin current (or spin motive force) in spintronics.

We are grateful to Z. S. Ma and Y. G. Yao for helpful discussions. This work was supported by NSF of China under grant 10347001, 90403034, 90406017, 60525417, 10665003, and by NKBRSF of China under 2005CB724508 and 2006CB921400.

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